

CS 240 Midterm Review

Question 1

True or False?

- a) If $\lim_{n \rightarrow \infty} \frac{f(n)}{g(n)} = e^{42}$ then $f(n) \in \Theta(g(n))$
- b) If $f(n) \in O(g(n))$ then $\lim_{n \rightarrow \infty} \frac{f(n)}{g(n)} = 0$
- c) If a rotation in an AVL tree is performed after an insertion, the height of the AVL tree is guaranteed to return to what it was before the insertion.
- d) We may have to do $\Omega(\log(n))$ rotations after insertion into an AVL tree
- e) The height of a binary search tree with n nodes is in $O(n)$.
- f) All binary heaps satisfy the AVL tree property
- g) There may be undiscovered comparison based sorting algorithms that are asymptotically faster than heapsort
- h) A skip list with n keys and height h will use $\Theta(nh)$ space.
- i) The most searched for item will eventually move to the front of the list if we use the transpose heuristic

Question 2

Order Notation

- a) Show that $3n^2 - 8n + 2 \in \Theta(n^2)$ from first principles
- b) Prove from first principles that $\frac{1}{2}n - 22 \in o(n \log(n))$
- c) Prove that $f(n) \in o(g(n)) \Leftrightarrow g(n) \in \omega(f(n))$

Question 3

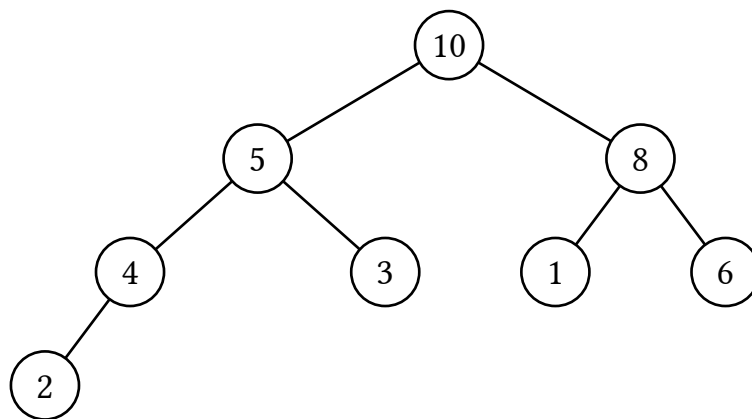
Algorithm Analysis

- a) `i = 2`
`x = 0`
`while i < n:`
 `for j = 1, ..., n:`
 `for k = 1, ..., j:`
 `x = x + 1`
 `i = i * i`
- b) `def isSorted(A, n = A.size): #find average runtime`
 `for i = 0, ..., n-2:`
 `if A[i] > A[i+1]: return False`
 `return True`

Question 4

Binary Heaps

- a) Insert 9 into the following max-heap and then delete-max



- b) Describe an algorithm for merging 2 binary heaps. That is, given 2 heaps A and B , return a new heap C containing all of the elements of A and B .

Question 5

Lower Bounds

Hex thinks that he has invented a comparison based implementation of the priority queue ADT which supports insert and delete-max in $O(\log(\log(n)))$ time. Explain why this cannot possibly be correct.

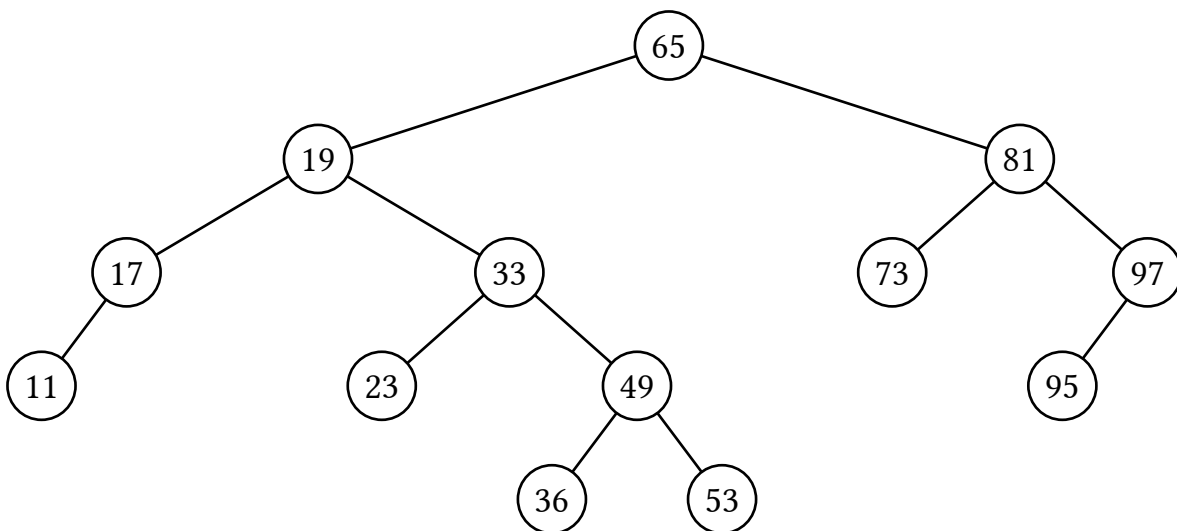
Question 6

Expected vs Average

Under what circumstances can we conclude that the randomized version of an algorithm has the same expected runtime as the average runtime of the original algorithm?

Question 7

Consider the following AVL tree



Suppose we are trying to insert a key into this tree

- What value of x would cause a left rotation?
- What value of x would cause a right rotation?
- What value of x would cause a double left rotation?
- What value of x would cause a double right rotation?

Question 8

Skip Lists

- a) Show that even for skip lists which have the expected number of keys at each level, it is possible that for some key, $\Omega(n)$ comparisons will be required during search. Do this by giving an example of such a skip list and key, and explain how this can generalize to arbitrarily large n .
- b) Insert the numbers 12, 11, 13, 10, 20 into an empty skip-list using the coin flips HHTHTHTTTHHHT. Then delete 13.

Question 9

Move to Front Heuristic

You are given a dictionary implemented with an array and the MTF heuristic. Suppose you have a sequence of n search queries where at most \sqrt{n} unique items were searched for. What is the best and worst case runtime to perform all searches?