11: Mutual Recursion

Mutual recursion

Mutual recursion occurs when two or more functions apply each other: `f` applies `g` and `g` applies `f`.

Mutually recursive functions can sometimes be merged into a single function. But using mutual recursion often leads to simpler code that is easier to understand.

Image: Drawing Hands, M.C. Escher
https://mcescher.com/

We'll look at three examples:

- mutual recursion on a natural number.
- mutual recursion on a list.
- mutual recursion on a tree.

Mutual recursion on Nats and Lists is rare. We're showing examples to ease into the pattern. Mutual recursion on general trees is common and a very useful tool.

Mutual recursion on a `Nat`

Consider the following observations about even and odd natural numbers:

- a number is even if it is one more than an odd number
- a number is odd if it is one more than an even number
- 0 is even (and therefore is not odd)

We can use these observations to implement `(is-even? n)`, which produces `true` if *n* is even and `false` otherwise.

is-even?


```
;; (is-even? n) produces true if n is even and false otherwise
;; Examples:
(check-expect (is-even? 4) true)
(check-expect (is-even? 5) false)

;; is-even?: Nat -> Bool
(define (is-even? n)
  (cond [(= 0 n) true]
        [else (is-odd? (sub1 n))]))

;; is-odd?: Nat -> Bool
(define (is-odd? n)
  (cond [(= 0 n) false]
        [else (is-even? (sub1 n))]))
```

Ex. 1

Alice and Bob are playing the game of "pebbles". There are *n* pebbles on a pile. Beginning with Alice, they alternate turns taking either one or two pebbles from the pile. The person who takes the last pebble wins.

Alice's strategy is to always take a single pebble. Bob's strategy is to take 1 pebble if there are an odd number and 2 pebbles if there is an even number.

Write `(pebbles n)` where `n` is the number of pebbles in play. The easiest approach is to use mutual recursion with one function implementing Alice's strategy and another function implementing Bob's strategy.

```
(check-expect (pebbles 1) "Alice wins")
(check-expect (pebbles 2) "Bob wins")
(check-expect (pebbles 3) "Bob wins")
```

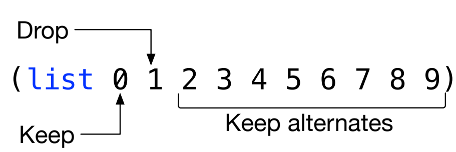
Mutual recursion on a list

We want to divide a list into two equal or nearly equally long lists.

If we had a function, `keep-alternates`, that keeps every other element of a list, we could apply it to the entire list and the rest of the list to divide it in two:

```
(check-expect (keep-alternates (list 0 1 2 3 4 5 6)) (list 0 2 4 6))
(check-expect (keep-alternates (rest (list 0 1 2 3 4 5 6))) (list 1 3 5))
```

Thinking about `keep-alternates`:



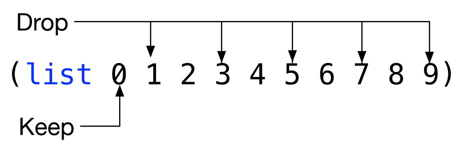
This suggests processing the list in two-element chunks. But that gets slightly messy with multiple base cases.

keep-alternates   M11 6/12

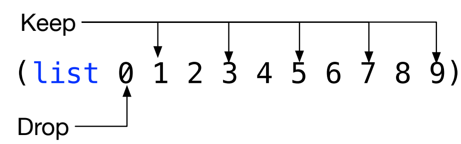
Another approach is that `keep-alternates` should keep the first element of the list and **drop** the alternating elements of the rest of the list.

How do we drop alternates? We keep the alternates from the rest of the list.

`keep-alternates`



`drop-alternates`



`keep-alternates` and `drop-alternates`   M11 7/12

```
;; (keep-alternates lst) keeps the first element of lst and drops
;; alternating elements from the rest.
;; Examples:
(check-expect (keep-alternates (list 0 1 2 3 4 5 6)) (list 0 2 4 6))
(check-expect (keep-alternates (rest (list 0 1 2 3 4 5 6))) (list 1 3 5))
(check-expect (keep-alternates (list 'a 'b 'c 'd)) (list 'a 'c))

;; keep-alternates: (listof Any) -> (listof Any)
(define (keep-alternates lst)
  (cond [(empty? lst) empty]
        [else (cons (first lst) (drop-alternates (rest lst)))]))

;; (drop-alternates lst) drops the first element of lst and keeps
;; alternating elements from the rest.
(define (drop-alternates lst)
  (cond [(empty? lst) empty]
        [else (keep-alternates (rest lst))]))
```

Mutual recursion on a tree   M11 8/12

Take a close look at our function for evaluating binary expressions:

```
;; eval: BinExp → Num
(define (eval ex)
  (cond [(number? ex) ex]
        [(binode? ex) (eval-binode ex)]))


(define (eval-binode node)
  (cond [(symbol=? '* (binode-op node))
         (* (eval (binode-left node)) (eval (binode-right node)))]
        [(symbol=? '/ (binode-op node))
         (/ (eval (binode-left node)) (eval (binode-right node)))]
        [(symbol=? '+ (binode-op node))
         (+ (eval (binode-left node)) (eval (binode-right node)))]
        [(symbol=? '- (binode-op node))
         (- (eval (binode-left node)) (eval (binode-right node)))]))
```

Self-referential data definitions

Mutual recursion arises naturally when the data definition references itself. Notice how a `BinExp` references a `BINode` and a `BINode` references a `BinExp`:

```
(define-struct binode (op left right))
;; A Binary arithmetic expression Internal Node (BINode)
;;     is a (make-binode (anyof '* '+ '/ '-) BinExp BinExp)

;; A binary arithmetic expression (BinExp) is one of:
;; * a Num
;; * a BINode
```

The mutual recursion will appear naturally in the templates, provided each part of the data definition is turned into a template.

`binexp-template` revisited

```
;; binode-template: BINode → Any
(define (binode-template node)
  (... (binode-op node)
       (binexp-template (binode-left node))
       (binexp-template (binode-right node))))

;; binexp-template: BinExp → Any
(define (binexp-template ex)
  (cond [(number? ex) (... ex)]
        [(binode? ex) (binode-template ex)]))
```

In our Nat and List examples, the two functions were very similar to each other. Here, we see one handling a `BinExp` and the other handling a `BINode`. In the next module a common pattern will be for one function to handle a tree and the other to handle a list of (sub)trees.

Goals of this module

- You should understand the idea of mutual recursion for examples given in lecture.
- You should be able to develop templates from mutually recursive data definitions, and to write functions using the templates.

The following functions and special forms have been introduced in this module:

You should complete all exercises and assignments using only these and the functions and special forms introduced in earlier modules. The complete list is:

`* + - ... / < <= = > >= abs add1` **`and`** `append boolean? ceiling char-alphabetic? char-downcase char-lower-case? char-numeric? char-upcase char-upper-case? char-whitespace? char<=? char<? char=? char>=? char>? char? check-error check-expect check-within` **`cond`** `cons cons? cos` **`define define-struct`** `define/trace e eighth` **`else`** `empty? equal? error even? exp expt fifth first floor fourth integer? length list list->string list? log max min modulo negative? not number->string number? odd?` **`or`** `pi positive? quotient remainder rest reverse round second seventh sgn sin sixth sqr sqrt string->list string-append string-downcase string-length string-lower-case? string-numeric? string-upcase string-upper-case? string<=? string<? string=? string>=? string>? string? sub1 substring symbol=? symbol? tan third zero?`