CS135 Tutorial 07

Binary Trees

In class we introduced binary trees with the following data-definition

```
(define-struct node (key left right))
;; A Node is a (make-node Nat BT BT)

;; A binary tree (BT) is one of:
;; * empty
;; * Node
```

Today:

- Recap on BT and BST
- Use the template to build functions for them
- Introduce the definition of balance
- Determine whether an existing tree is balanced
- Build a balanced tree

Binary Tree Template

We also used that data definition to come up with bt-template

```
;; bt-template: BT →Any
(define (bt-template bt)
   (cond
      [(empty? bt) ...]
      [(node? bt) (... (node-key bt)
                       (bt-template (node-left bt))
                       (bt-template (node-right bt)))]))
```

bt-size

Write a function bt-size to count the number of nodes in a binary tree.
Let’s use the template!!!

Binary Search Tree (BST)

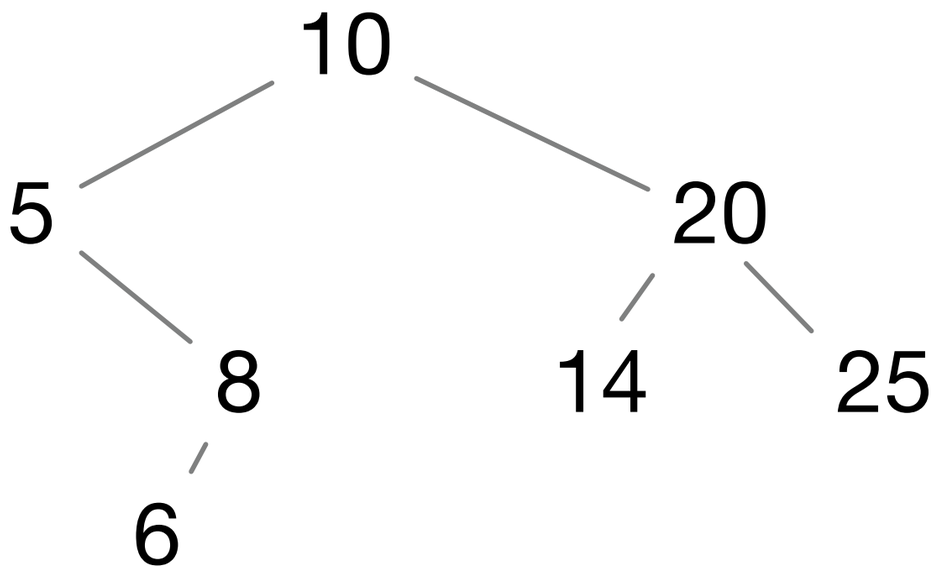
We also introduced binary search trees with the following data definition:

```
;; A Binary Search Tree (BST) is one of:
;; * empty
;; * a Node

(define-struct node (key left right))
;; A Node is a (make-node Nat BST BST)
;; Requires: key > every key in left BST
;; key < every key in right BST
```

Count-range

Write a function, `(count-range bst lo hi)`.
It produces the number of keys in *[lo, hi]* – between `lo` and `hi`, inclusive.



For the BST shown:

```
(check-expect (count-range bst 0 50) 7)
(check-expect (count-range bst 11 25) 3)
(check-expect (count-range bst 8 8) 1)
(check-expect (count-range bst 11 13) 0)
```

Use the ordering property of BSTs.

Balanced Binary Trees

- In class we've also commented that sometimes there are advantages to a "balanced" binary tree – especially when searching a BST
- There are several definitions of "balanced". Here's one:

- A binary tree is balanced if:
  - The number of nodes in the left and the right subtrees differ by at most 1
  - Both subtrees are also balanced.
  - An empty tree is balanced.

Balanced BST Data Definition

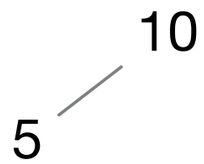
```
(define-struct node (key left right)
;; A Node is a (make-node Nat BalBST BalBST)
;; requires: all keys in left < key
;;           all keys in right > key
;;           |(# nodes in left) - (# nodes in right)| <= 1

;; A balanced binary tree (BalBST) is one of:
;; * empty
;; * Node
```

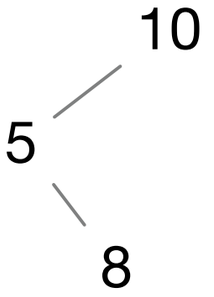
(balanced? bt)

- A binary tree is balanced if:
  - The number of nodes in the left and the right subtrees differ by at most 1
  - Both subtrees are also balanced.
  - An empty tree is balanced.

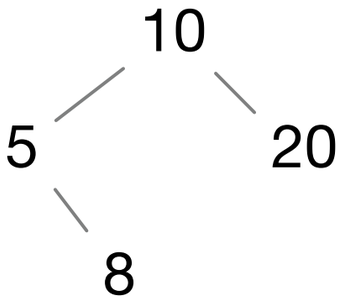
We are *not* checking if the BST ordering property holds.

`(check-expect (balanced? empty) ...)`	
`(check-expect (balanced? (make-node 10 empty empty)) ...)`	

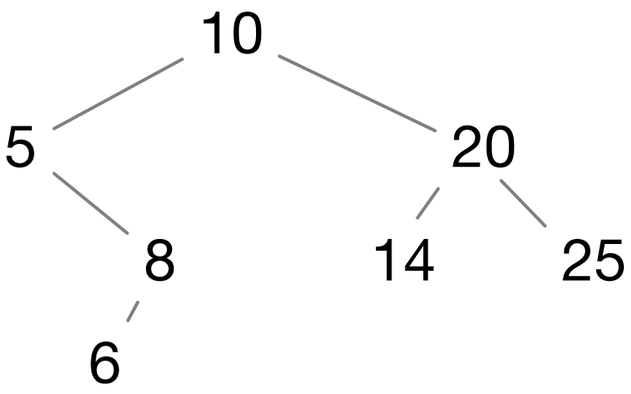
```
(check-expect
 (balanced?
  (make-node 10
       (make-node 5 empty (make-node 8 empty empty))
       empty))
 ...)
```




```
(check-expect
 (balanced?
  (make-node 10
       (make-node 5 empty (make-node 8 empty empty))
       (make-node 20 empty empty)))
 ...)
```




```
(check-expect
 (balanced?
  (make-node 10
       (make-node 5 empty (make-node 8
                            (make-node 6 empty empty) empty))
       (make-node 20 (make-node 14 empty empty)
                     (make-node 25 empty empty))))
 ...)
```



Building balanced binary search trees

Given a sorted list of number, build a balanced binary search tree.

```
(define-struct node (key left right))

;; (build-bal-bst slon) builds a balanced binary search tree from slon.
;; build-bal-bst: (listof Num) -> BalBST
;; requires: slon is sorted in increasing order
(define (build-bal-bst slon) ...)

(check-expect (build-bal-bst empty) empty)
(check-expect (build-bal-bst (list 1)) (make-node 1 empty empty))
(check-expect (build-bal-bst (list 1 2 3 4 5 6))
     (make-node 4
          (make-node 2 (make-node 1 empty empty) (make-node 3 empty empty))
               (make-node 6 (make-node 5 empty empty) empty)))
```

Required helper functions

```
;; (nth-elem lst n) produces the nth element in lst (counting from 0).
;; nth-elem: (listof X) Nat -> X
(define (nth-elem lon n)
  (cond [(zero? n) (first lon)]
        [else (nth-elem (rest lon) (sub1 n))]))

;; (take lon n) produces a list from the first n elements f lst.
;; take: (listof X) Nat -> (listof X)
(define (take lon n)
  (cond [(zero? n) empty]
        [else (cons (first lon) (take (rest lon) (sub1 n)))]))

;; (drop lon n) produces a list from the elements after the first n+1 elements
(define (drop lon n)
  (cond [(zero? n) (rest lon)]
        [else (drop (rest lon) (sub1 n))]))
```

Required helper functions

```
(define lst (list 0 1 2 3))

(check-expect (nth-elem lst 0) 0)
(check-expect (nth-elem lst 1) 1)
(check-expect (nth-elem lst 3) 3)
(check-expect (take lst 0) empty)
(check-expect (take lst 1) (list 0))
(check-expect (drop lst 0) (list 1 2 3))
(check-expect (drop lst 1) (list 2 3))
(check-expect (drop lst 3) empty)

(check-expect (append (take lst 0) (list (nth-elem lst 0)) (drop lst 0)) lst)
(check-expect (append (take lst 1) (list (nth-elem lst 1)) (drop lst 1)) lst)
(check-expect (append (take lst 2) (list (nth-elem lst 2)) (drop lst 2)) lst)
(check-expect (append (take lst 3) (list (nth-elem lst 3)) (drop lst 3)) lst)
```